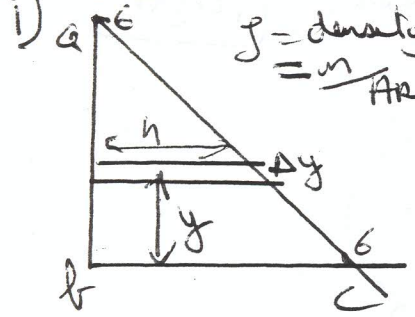
1983 MofI :

i) 

ρ = density $= \frac{m}{\text{Area}}$

$\Delta I = \Delta m y^2$

$= \rho(\text{area}) y^2$

$= \rho (h \Delta y) y^2$

$\Rightarrow I_{bc} = \int_0^6 \rho (6-y) y^2 dy$

$= \rho \int_0^6 (6y^2 - y^3) dy$

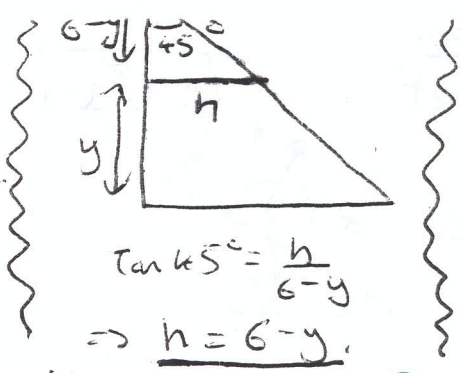
$= \rho \left[2y^3 - \frac{y^4}{4} \right]_0^6$

$= \rho \left(2(6)^3 - \frac{6^4}{4} \right)$

$= \rho (432 - 324)$

$= 108 \rho = 6m$

$I_{ab} = I_{bc}$ by symmetry



$\tan 45^\circ = \frac{h}{6-y}$

$\Rightarrow h = 6 - y.$

mass $= \rho$ area

$= \rho \frac{1}{2} 6 \cdot 6$

$= \rho 18$

---

i) Find I_a MUST "GO THROUGH g (centre of gravity)

$I_b = I_{ab} + I_{bc} = 6m + 6m = 12m.$ (⊥ axes theorem)

$I_g = ?$ gar(2,2)

$I_b = I_g + m|gb|^2 \Rightarrow 12m = I_g + m[(0-2)^2 + (0-2)^2]$

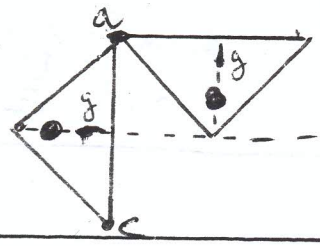
$\Rightarrow 12m = I_g + 8m$

$\Rightarrow \boxed{I_g = 4m}$

$I_a = I_g + m|ag|^2$

$I_a = 4m + m[(0-2)^2 + (6-2)^2] = 4m + m(20) = 24m$ $\boxed{I_a = 24m}$

---

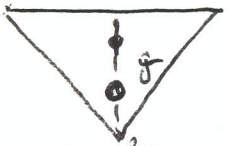
iii) 

$\mathcal{E}_A = \frac{1}{2} I \omega^2 + mg\bar{h}$

$\Rightarrow \mathcal{E}_A = \frac{1}{2} I (0)^2 + mg\bar{h}$ (*)

$\Rightarrow \mathcal{E}_A = 0 + mg\sqrt{8}$

(*) Find $\bar{h}$ for Pos(A)



FROM ABOVE.

$|bg| = \sqrt{(2-0)^2 + (2-0)^2}$

$= \sqrt{8}.$

$\mathcal{E}_B = \frac{1}{2} I \omega^2 + mg(0)^2$

$\Rightarrow \mathcal{E}_B = \frac{1}{2}(24m)\omega^2$

$\Rightarrow \mathcal{E}_B = 12m\omega^2.$

Find ω:

$\mathcal{E}_A = \mathcal{E}_B \Rightarrow 12m\omega^2 = mg\sqrt{8}$

$\Rightarrow \omega = \sqrt{\frac{\sqrt{8}}{12} g}$ Rad/sec

(*) Find r. FROM PYTHAG.

$r = |ac| = \sqrt{6^2 + 6^2}$

$= \sqrt{72}$

Find v

$v = r\omega \Rightarrow v = \sqrt{\frac{\sqrt{8}}{12} g} \cdot \left[\sqrt{72} \right]$ (**)

$\Rightarrow v = \sqrt{12\sqrt{2} g} = \sqrt{3\sqrt{2} g}$ ms^{-1}

LONG QUESTION !